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Experimental Study of Cubic, Pyramidal and Hemispherical Diffusers at Normal Sound Incidence

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ABSTRACT

For successful room acoustic design, it is important to know sound scattering properties of different types of rough surfaces. Both relief surfaces and single diffusers on the flat surface are commonly used in practice. In this work we consider the single scattering elements and the array of the diffusers. If the distance between the elements is much greater their sizes they can be assumed as single elements. When the elements are close to each other they form the array. So the number of elements per unit area is an important parameter determining the scattering coefficient, which also depends on the form of the diffusers and the angle of incidence of the sound wave. In many cases sound waves should be diffused under normal incidence and here we focus on the scattering properties of the surfaces under normal sound incidence. Recently proposed method for measurement of scattering coefficients in a non-diffuse sound field is used for experimental investigation. Highly anisotropic sound field is created in a rectangular room with two absorbing non-parallel walls. Tested sound scattering elements are placed on the rigid walls perpendicular to the absorbing surfaces. The walls with the elements are named the tested walls. Sound waves propagate along the absorbing walls and normal to the tested walls. Cubic, pyramidal and hemispherical diffusers of similar sizes are investigated for different number of diffusers per unit area. It is found that the cubic diffusers are the most effective when their number is not great. If the diffusers completely cover the tested surface an array of the pyramids scatters maximal sound energy.

Keywords: scattering coefficient, normal sound wave incidence, sound decay, rectangular room.

1. INTRODUCTION

There are several methods for measuring the scattering characteristics of sound waves by relief surfaces and individual scatterers [1,2]. Recently proposed method for measurement of scattering coefficients in a non-diffuse sound field is used for experimental investigation. The scattering coefficient is determined by the decay curve defined by the exponential power function and measured in a rectangular room with an uneven distribution of absorption on its walls. To measure the scattering coefficient, conversely, a non-diffused sound field is required, which energy decay law differs maximally from the exponential curve. Such a field can be generated, e.g., in a room shaped like a rectangular parallelepiped with an uneven distribution of absorption on the walls. The theoretical basis of the new method and the results of experiments verifying it are presented in [3-5]. In this work the method is developed for measuring the scattering coefficient under normal incidence of a sound wave on a scattering surface.

2. SOUND DECAY IN A RECTANGULAR ROOM

The decay of sound energy in a rectangular room (Fig. 1) with smooth walls at high frequencies is determined by the law found in [6]:

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$$E_s(t) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} e^{\frac{t}{2}\sum_{i=1}^{6} n_i \ln\gamma_i} \cos\theta d\theta d\varphi \tag{1}$$

where t is time; i is the number of the room wall;

$$n_{1,2} = \frac{c}{2H}\sin\theta, \qquad n_{3,4} = \frac{c}{2D}\cos\theta\sin\varphi, \qquad n_{5,6} = \frac{c}{2L}\cos\theta\cos\varphi;$$

where H, L, D are the lengths of the sides of the room; $\gamma_i = 1-\alpha_i$ and α_i are reflection and absorption coefficients of the *i*-th wall.

According to (1) the sound energy in a room is the sum of the energies carried by the rays, which directions are given by the angles θ and φ. With time the energy of each ray decreases because of reflections from the absorbing walls, i.e. for which $\alpha_i > 0$.

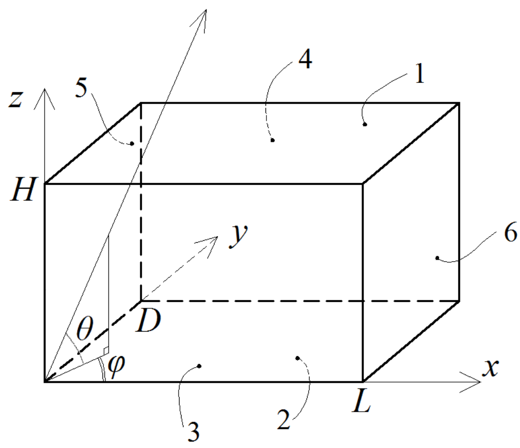


Figure 1 – A rectangular room

Let walls 1 and 3 have an acoustic impedance *Z*, then their reflection coefficients are equal [6]

$$\gamma_1(\theta,\varphi) = \left|\frac{Z\sin\theta - 1}{Z\sin\theta + 1}\right|^2, \qquad \gamma_3(\theta,\varphi) = \left|\frac{Z\cos\theta\sin\varphi - 1}{Z\cos\theta\sin\varphi + 1}\right|^2. \tag{2}$$

If the remaining walls are reflective, i.e.,$\gamma_2 = \gamma_4 = \gamma_5 = \gamma_6 = 1$, then the decay law for $t >> \frac{D}{c}, \frac{H}{c}$ takes the form $E_s(t) \sim 1/t$ [6].

During the reverberation process sound rays are reflected by the walls specularly, therefore the sound energy is often called the energy of specular reflections. If some walls of the room are relief, a part of the sound energy is scattered. To quantify the scattering properties of the walls, along with the specular reflection coefficient γ and absorption coefficient α, a scattering coefficient is introduced δ ,which is the ration of the scattered energy to incident one. These coefficients are related by the law of conservation of the energy $\alpha + \delta + \gamma = 1$.

Suppose that in a room with two absorbing walls the third wall that is not parallel to them is scattering. Its absorption coefficient $\alpha_6 = 0$, and the scattering coefficient δ_6 in the general case depend on the incidence angle of the wave.

The specular reflection coefficient is determined from the law of conservation of energy

$$\gamma_6(\theta,\varphi) = 1 - \delta_6(\theta,\varphi). \tag{3}$$

Let three walls to be smooth and absolutely rough, i.e. $\gamma_2 = \gamma_4 = \gamma_5 = 0$, we substitute (2) and (4) into (1) and find the decay law for $t >> \frac{L}{c}, \frac{D}{c}, \frac{H}{c}$

$$E_s(t) \approx \frac{\sqrt{HD}}{4ct \operatorname{Re} Z} e^{-\frac{ct}{2L}\delta_6(0,0)} \tag{4}$$

So the decay law depends on the normal scattering coefficient of the wall with the number $i = 6$, and the value of this coefficient can be determined from the measured decay curve by means of an approximation by function (4).

The obtained result has a simple physical interpretation. The initial distribution of energy carried along the rays in the corners θ and φ is uniform. First of all the energy of the rays incident on the absorbing wall at large angles is absorbed.

The sound field becomes quickly anisotropic. The energy of the rays propagating along the absorbing walls, i.e. normal to the scattering wall is dissipated slowly. It takes place because, firstly, these rays are rarely reflected from the absorbing walls, and secondly, the absorption coefficient at a sliding incidence on impedance walls is close to zero. The decay rate of the energy of these rays depends on the scattering coefficient of the third wall: the higher it is, the more energy is dissipated and gets to the absorbing walls not at sliding angles. The scattered energy is rapidly absorbed, which makes the effect of absorption and scattering of the wall on the sound decay law equivalent [4].

The total sound energy in the room is determined by the energy of specular reflections (4) and the scattered energy. Let us estimate the latter as follows. There is only one wall in the room $i = 6$ with non-zero scattering coefficient δ_6. By time wave whose direction is characterized by angles φ and θ, has $N = t_0 \sum_{i=1}^{6} n_i(\theta,\varphi)$ specular reflections. The energy of specular reflections before the last reflection is determined by the first $N-1$ reflections. If N-th reflection comes from the wall $i = 6$, then the energy scattered by the N-th reflection is:

$$\delta_6 \gamma_6^{\frac{t_0}{2} n_6(\theta,\varphi)-1} \prod_{i \neq 6} \gamma_i^{\frac{t_0}{2} n_i(\theta,\varphi)} = \frac{\delta_6}{\gamma_6} \prod_i \gamma_i^{\frac{t_0}{2} n_i(\theta,\varphi)} = \frac{\delta_6}{\gamma_6} e^{\frac{t_0}{2}\sum_i n_i(\theta,\varphi)\ln\gamma_i} \quad ; \tag{5}$$

During the time interval from t_0 to $t_0 + dt_0$, $n_6 dt_0$ reflections from the scattering wall occur. To obtain the scattered energy for this interval, we integrate over the angle:

$$dE_d = \frac{dt_0}{4\pi} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} n_6(\theta,\varphi) \frac{\delta_6(\theta,\varphi)}{\gamma_6(\theta,\varphi)} e^{\frac{t_0}{2}\sum_i n_i(\theta,\varphi)\ln\gamma_i} \cos\theta d\theta d\varphi \, ; \tag{6}$$

At the moment of time t_0 the scattered energy is equal dE_d. It decays according to the law (1), therefore, by the time moment t it is $E_s(t-t_0)dE_d$. To obtain the scattered energy in the room by the time moment t, we integrate (6) over time from the beginning of the reverberation process:

$$E_d = \frac{1}{4\pi} \int_0^t dt_0 E_s(t-t_0) \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} n_6(\theta,\varphi) \frac{\delta_6(\theta,\varphi)}{\gamma_6(\theta,\varphi)} e^{\frac{t_0}{2}\sum_i n_i(\theta,\varphi)\ln\gamma_i} \cos\theta d\theta d\varphi \quad ; \tag{7}$$

Substituting (4) into (7), we find an approximate expression for the scattered energy

$$E_d(t) \approx \frac{\pi^2}{64} \frac{\delta_\perp}{1-\delta_\perp} \frac{HD}{ctL \operatorname{Re} Z} e^{-\frac{ct}{2L}\delta_\perp} \, ; \tag{8}$$

where $\delta_\perp = \delta_6(0,0)$.

The energy of specular reflections (4) and the scattered energy (8) depend on time equally, therefore the dependence of the total sound energy in the room $E = E_s + E_d$ on time has the form

$$E(t) \approx \frac{a}{t} e^{-bt} . \tag{9}$$

So the measured decay curve can be approximated by dependence (9), the coefficients *a* and *b* are determined, and the scattering coefficient is found from the expression:

$$\delta_{\perp} = \frac{2L}{c} b . \tag{10}$$

Note that we considered only a single sound scattering. After the next reflection, the scattered rays will again fall on the relief wall, while the scattered energy will also add to the total sound energy in the room, unaccounted for in (8). The value of this energy can also be estimated as the energy of single scattering (7), it is obvious that it will be proportional $\delta_{\perp}^{2}$. Multiple scattering energy can be introduced in a similar way, and it will be proportional $\delta_{\perp}^{m}$, where m - number of reflections from the scattering wall.

The energy E_d, taking into account all the scatterings decays according to the law (9) as well, so it would be possible to conclude that the sound decay in the room occurs by to the same law for any values of the scattering coefficient. However, (9) was obtained with the relation (4), is right for $t >> \frac{L}{c}$, i.e. we assume that the scattering on the wall is small, so the scattered energy does not have time to quickly absorb, and its decay occurs according to the law (9).

If a significant part of the energy is scattered upon reflection from the relief wall, then, firstly, the energy of specular reflections E_s decreases rather quickly, and secondly, the decay of the scattered energy occurs according to the law (1) with $t \sim L/c$, which differs from the approximate decay law (4). In the present work, the case of large scattering coefficients is not considered, but in the limiting case, the decay law should go over to the usual Sabin's law [7].

3. SCALE MODEL EXPERIMENT

The proposed method for measuring the sound scattering coefficient at normal incidence was tested in a scale-model experiment. In a rectangular room (Fig. 2) with dimensions 0.7x0.4x0.4 m, two non-parallel long walls are covered with sound-absorbing material. One of the walls with dimensions 0.4x0.4 m is designed to place the tested sound diffusers. In one of the corners of the room there is a sound source with a sufficiently wide radiation pattern to consider the sound field isotropic at the initial time. The decay curve is measured with an omnidirectional microphone located near the test wall. Note that the decay curves measured at different points do not differ, therefore the averaging of measurements over space was not performed.

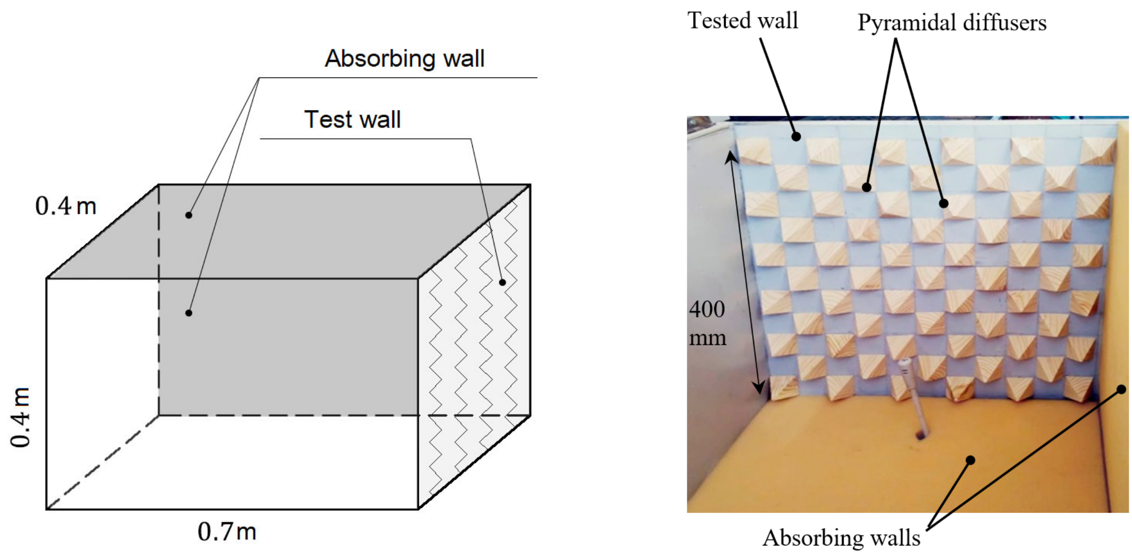


Figure 2 – A scale-model room

First measurement was carried out without diffusers. Using the decay curve measured in such a room as an example, we show the procedure of approximation. In Fig. 3a the measured decay curve in the octave band 8 kHz is shown. For analysis we select its part in the time interval from t_1 to t_2. The lower bound taken from the relation $t >> \frac{L}{c} = 2$ ms. The value t_2 is according to the point in time when the value of the decay curve is 10 dB higher than the background noise. Thus, the approximation of the measured curve is performed on a time interval limited by the values $t_1 = 20$ ms and $t_2 = 196$ ms. Curve (9) with coefficients $a = 4.7 \cdot 10^{-4}$ and $b = 37.0$, which values are selected by the least squares method, is shown in dotted line in Fig. 3a.

Without diffusers, the test and opposite walls have the same acoustic properties. The found value b can be used to determine the absorption coefficient of the walls α. In the left-hand side (10) instead $\delta_\perp$ it is necessary to substitute the sum of the absorption coefficients of these walls, i.e. 2α, then the sound absorption coefficient of the test wall is $\alpha = Lb/c = 0.078$, where $L = 0.7$ m.

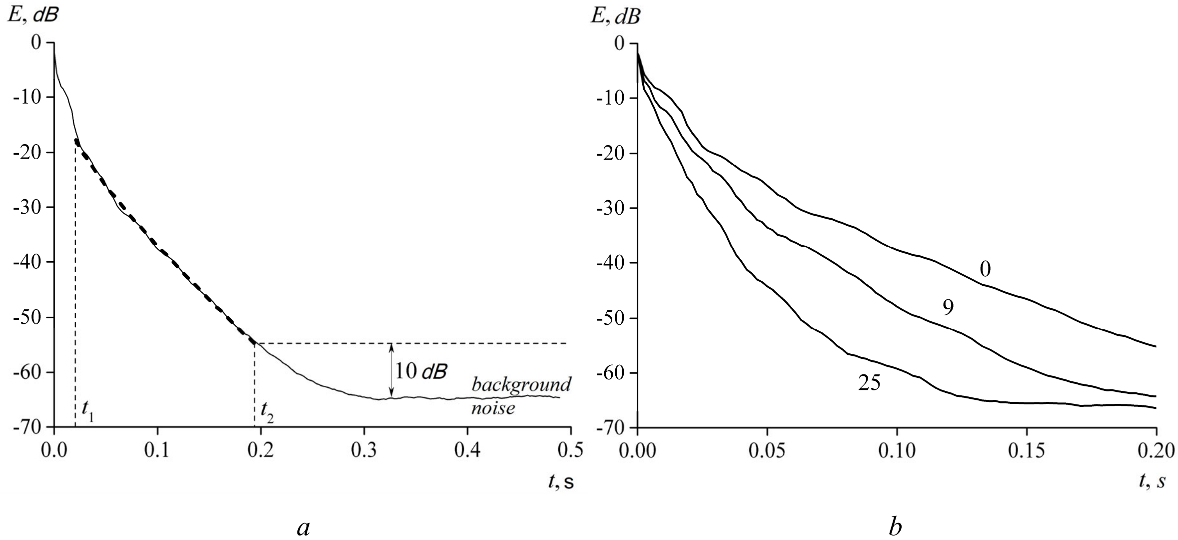


Figure 3. The measured decay curves in the model experiment (Fig. 2): *a* - an example of approximation of the measured decay curve by the theoretical dependence (9), indicated by a dotted line; *b* - decay curves measured in a room without scattering cubic elements (curve 0), as well as with 9 and 25 elements.

In practice the test surface always absorbs sound. Sound absorption can be quite substantial, especially at high frequencies. If $\alpha, \delta << 1$, then the effect of sound absorption and scattering by the test wall on the decay law is the same [4], therefore, instead of the scattering coefficient $\delta_6(0,0)$ of the non-absorbing surface, in (4), we must substitute the sum of the absorption and scattering coefficients of the real test wall $\delta' = \delta_6(0,0) + \alpha_6(0,0)$. For further calculation of the scattering coefficient, it is necessary to take into account the absorption coefficient measured in a room without samples.

In the experiment, wooden cubes, hemispheres and pyramids with a side of 3.5 cm were used as diffusers (Fig. 4), which were glued to the surface of the test wall (Fig. 2). Note that such relief surfaces are often used for experimental studies [8-9].

4. MEASUREMENT RESULTS

In Fig. 5 shows the results of measuring the total absorption and scattering coefficient δ' of the test wall in octave bands 4 kHz and 8 kHz depending on the number of elements n on the test wall. Note that the scattering coefficient of the wall without elements (n = 0) is not zero because the wall slightly absorbs sound and the coefficient a in (9) is not zero as well. 121 elements completely cover the surface of the tested wall.

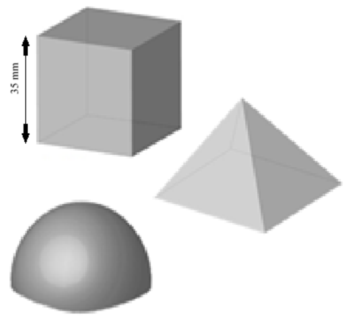


Figure 4 – Tested diffusers

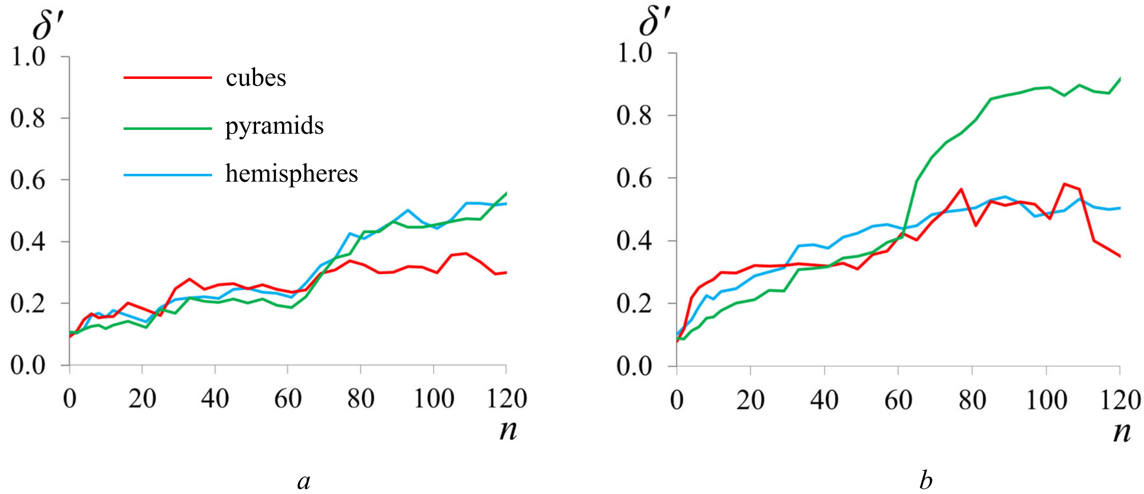


Figure 5 – Measured coefficient δ' at 4kHz (*a*) and 8 kHz (*b*) in dependence on the number of the diffusers *n*

On the graph (Fig. 5) we observe an increase in the coefficient δ' with an increase in the number of cubic diffusers.

At 4 kHz all elements have similar scattering coefficient, only with the number of diffusers $n > 80$ the cubes scatter sound weaker. The scattering surfaces formed by the hemispheres and pyramids ($n = 121$) have the scattering coefficient about 0.5.

At 8 kHz the tested elements have different scattering properties. When the number of diffusers is small ($n < 20$) the cubes scatter sound much better than the pyramids and hemispheres. The scattering coefficient of the pyramids increases with their number, whereas the cubes and hemispheres have a limit for the coefficient δ'. It does not exceed 0.5.

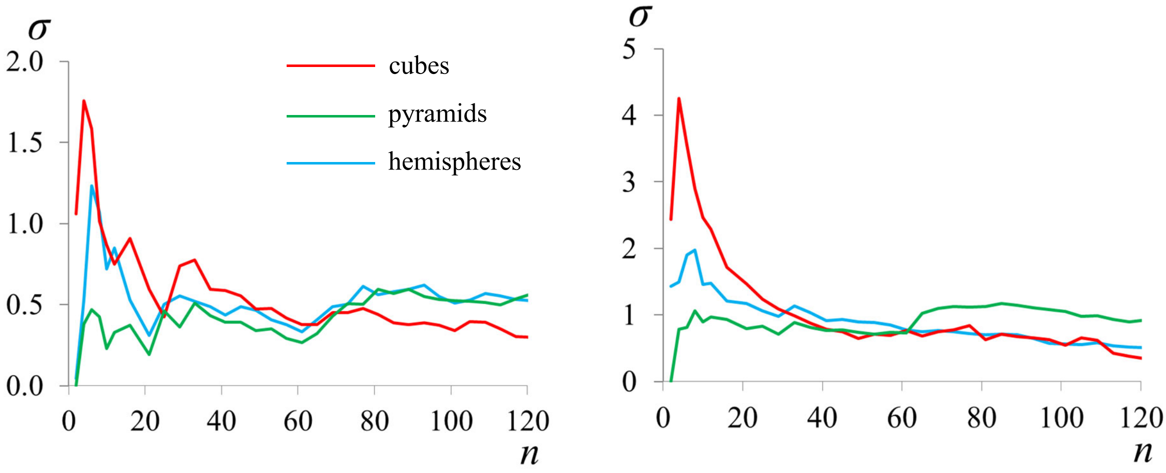
To characterize sound scattering properties of a single element let us introduce a dimensionless scattering cross section given by

$$\sigma(n) = \frac{\delta'(n)S - \delta'(0)(S - S_0 n)}{S_0 n}. \qquad (11)$$

where S is the area of the tested wall, S_0 is the area occupied by the element (square 3.5x3.5 sm), $\delta'(0)$ is the absorption coefficient of the tested wall. The scattering cross section calculated form the measured coefficient δ' is shown in Fig. 6.

The scattering cross section is maximal if there are 4-8 elements on the tested walls. Its values reach 1.7 at 4 kHz and 4.2 at 8 kHz.

In case of scattering surfaces ($n > 60$) the cross section is about 0.5 at 4 kHz for all types of the diffusers and at 8 kHz for the pyramids and hemispheres. It is equal 1 for pyramids.



a *b*

Figure 6 – Scattering cross section of one diffuser at 4kHz (*a*) and 8 kHz (*b*) in dependence on the number of the diffusers *n*

5. CONCLUSIONS

An experimental method for measuring the scattering coefficient under normal incidence of a sound wave on a relief surface is proposed. The measurement is carried out in a special room in the form of a rectangular parallelepiped; two non-parallel walls of the room are covered with absorbing material. The scattering coefficient of the test surface is found by approximating the sound decay curve measured in the room with a known theoretical dependence. The proposed method was tested on a scale model of a rectangular room using cubes, pyramids, and hemispheres diffusers on a flat surface.

It is found that the cubic diffusers are the most effective when their number is not great ($n < 30$), whereas the pyramids are the least effective. If the diffusers completely cover the tested wall ($n = 120$) the array of the pyramids scatters maximal sound energy. The scattering coefficient of the pyramids is close to 1, while the scattering coefficients of the cubes and hemispheres are less 0.5.

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