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Geometrical Computation of Arbitrary Curved Surface Sound Reflected Impulse Responses

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[1] Pachyderm Acoustical Sciences, Acoustic Distinctions

ABSTRACT

A practical method is presented for the calculation of reflection magnitude from curved surfaces in the context of acoustical simulation using NURBS models (for which minimum and maximum curvatures are known) backed with polygon approximations of geometry (a common means of geometrical representation in computer aided design). The new method is similar in approach to the Biot- Tolstoy-Medwin method of edge diffraction, but also incorporates a method developed by DesChamps in the 1970's (and later applied to acoustics by Pierce) to correctly calculate the effect of the reflecting surface on the radii of the wavefront. First, specular reflections from the surface are found using the image source method and edge querying techniques. The extent of the surface which reflects specularly is determined by clustering reflections in an appropriate way - for convex surfaces, a point - for cylindroid sections, a line or curve - for spheroid surfaces, a polygon approximation of the surface between reflecting points. Once the reflecting extent is found, an explicit continuous-time integral is calculated for the reflecting path in terms of root-mean-square pressure using DesChamps' method. The impact of phase is determined by convolving this function with a pulse of the correct time and frequency characteristics.

Keywords: Simulation, Curvature, Room Acoustics

1. INTRODUCTION

Traditionally, geometrical acoustics prediction techniques (as well as simulation in many other disciplines) rely heavily on the inverse square law of intensity for the prediction of spherical sound propagation. This technique works fine for most applications, if the wave-front never deviates from a spherical curvature. Unfortunately, there are many objects that can change the shape of a wave-front's curvature. Edges are one case. In Peter Svensson's formulation of the Biot-Tolstoy-Medwin method for edge diffraction (5), the change in wave-front shape is accounted for using Huygen's principle – a wave-front can be constructed using many small point sources along its surface. Using many spherical secondary sources with strength calculated based on ingoing and outgoing angle relative to the wedge, the method constructs the wave diffracted by an edge – each source propagating according to inverse distance law in pressure domain (the square of which is proportional to inverse square law in intensity). The product of this method is a root-mean-square pressure profile (which he refers to as impulse responses) which is then convolved with a signal comprised of the power and frequency content of the initial sound source.

Concave and convex curves will also alter a reflecting wave-front. In the case of reflections from curved surfaces, the inverse square law breaks down. Intensity may actually increase or decrease more quickly with distance following reflection. The method discussed here addresses this specific case with an aim to produce reliable deterministic geometrical acoustics calculations of rooms with curved geometry. It is similar to Svensson's formulation in that it produces root mean square pressure based "impulse responses" for sound reflections with a sustained presence in the time-domain. It differs in that it does not rely on a Huygens-style inverse distance construction of the wavefront, but instead calculates the curvature of the wave-front in order to predict the intensity (and resulting rms pressure) of the reflection impulse response.

2. THEORY

2.1 Previous Work

The most recent comprehensive treatment of curved surface reflection that we are aware of is Vercammen's PHD thesis (1), in which he makes a complete comparison of existing methods with regard to accuracy in simulation of focused sound reflections. One determination he makes is that the image-source method does a poor job predicting sound reflections. This is not surprising, as there are several ways that the image source method falls short for this purpose. 1) Whether or not it finds a sufficient number of paths depends on the model of the space. It is very likely that it would miss important paths, or even include too many (because it is not accounting for the area contributing, but dumbly adding them all together). 2) The equations for the traditional image source method are designed for reflections from flat surfaces. They do not make considerations for what happens to the shape of the wave-front following reflection. Despite these reasons, we would like to be clear that a deterministic geometrical method can be designed which can be used alongside the image source method in order to properly process reflections from curved geometry.

In the 1980's, several prominent researchers in acoustics published methods for calculating reflections from curved surfaces. Jens Holger Rindel published an empirical method which he related back to his 'characteristic distance' concept. (2) This method predicted the power of a reflection with very few variables, and little computation. While this would have been very useful for the time, desktop computers are now able to leverage greater computational resources and perform more sophisticated calculations with much more precise results. There is the potential to predict the entire reflection structure of a curved surface and apply phase, using a more physical method.

Ironically, methods that could achieve this were being studied a decade earlier in the field of electromagnetics. This work is summarized very neatly in the work of Deschamps (3), and later by Allen Pierce (4) (again in the 1980's) as he applies these methods to acoustics. In both cases, the method was used to predict intensity only. However, as exemplified by Deschamps' 1972 paper, the method can also be applied with sensitivity to phase - although the technique used here is slightly different (bearing more resemblance to the Biot-Tolstoy-Medwin-Svensson (5) method of edge diffraction impulse response construction).

2.2 Inverse Square Law as an Over-simplification of Wave-front Curvature

The nearly ubiquitous inverse-square law of intensity is traditionally used to predict the power at a given distance from a spherical emitter. It can be used no matter how many sound reflections the sound has encountered by adding the length of each leg of the sound's journey to the total radius, which is technically correct as long as nothing upsets the shape of the wave-front. In the equation below, I is the intensity at a distance r from the source, and W is the source power.

$$I = \frac{W}{4\pi r^2} \tag{1}$$

When a curved surface becomes the reflecting object, the wave-front may take on a different propagation form. The impact of the curved surface on the wave-front can be predicted by considering the curvature of the wave-front. This is typically represented by the curvature K, which can be found for any given curve in terms of its minimum principal curvature and maximum curvature, along with the respective tangent vectors along the radii. (See Figure 1, below)

The curvature K is the inverse of the radii, and is typically stored in a 2x2 curvature Tensor that can be rotated in order to apply the data to a variety of tangent spaces. (3) (4)

$$[K] = \begin{bmatrix} \frac{1}{r_a} & 0 \\ 0 & \frac{1}{r_b} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \tag{2}$$

The determinant of this tensor is always the gaussian curvature ($1 / r_a r_b$), regardless of rotation which is identical to the inverse square radius term of inverse-square law for the case of a spherical wave. Deschamps uses the curvature as a means for predicting the intensity of a wave-front in his 1972 paper, effectively as a replacement for inverse-square law.(3) For a spherical wave, either case could be used, and for any other wave-form, the curvature tensor $[K]$ would yield a more complete result.

$$\frac{W}{4\pi r^2} = \frac{W}{4\pi} * |det[K]| \tag{3}$$

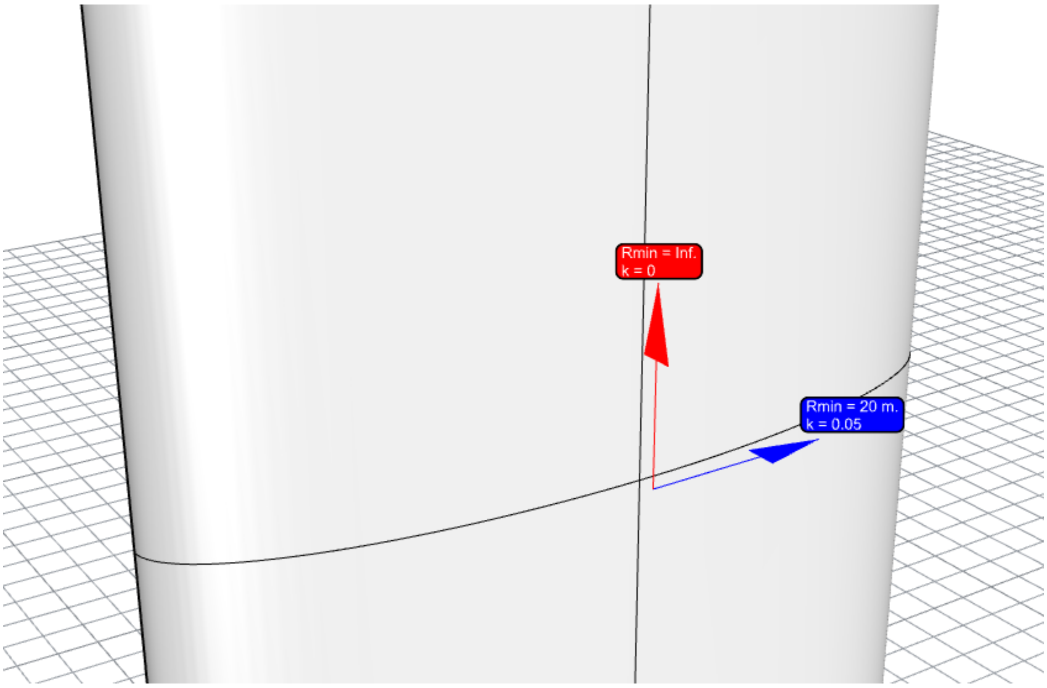

Figure 1 – A typical curvature representation of a curved surface. This surface exhibits convex curvature in one tangent direction, similarly to a cylinder.

2.3 Wave-front Curvature Tracing

Every surface in a room has a curvature tensor similar to that of the wave-front. A flat surface has zero curvature in both directions. A concave surface has at least one principal axis with negative curvature. A convex surface will have at least one positive non-zero radius and no negative radii. A comprehensive handling of wave-front curvature will be able to account for all of these cases.

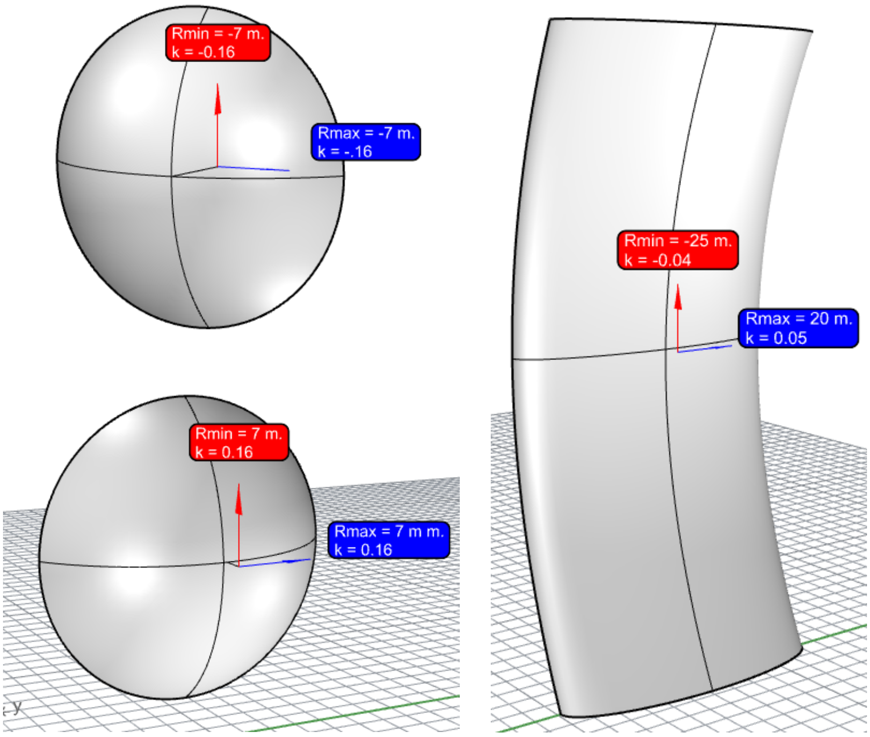

Figure 2 –Different forms of curvature: (upper left) a fully concave surface, (lower left) a fully convex surface, and (right) a saddle shaped surface with mean negative curvature.

A sound wave from a point source initially has equal curvature in both directions. This remains the case as long as it only reflects from surfaces with zero curvature. When a wave encounters a surface with curvature, the waveform is modified by the surface curvature. In order to calculate the modification, the wave-front and surface curvature tensors must be rotated to have at least one principal tangent vector coincide. In order to find the angle of rotation for each Tensor, you find the line of intersection of the planes perpendicular to the surface normal at the point of incidence, and the plane perpendicular to the direction of sound (shown in purple in figure 3 below). This line is effectively the common vector of both planes. The tensors for the surface and the wave-front need to be rotated so that one of the principal radii coincides with this vector. The inverse cosine of the dot product of the tangent vector to the principal radii of each curvature tensor and the line of intersection gives you the angle of rotation.

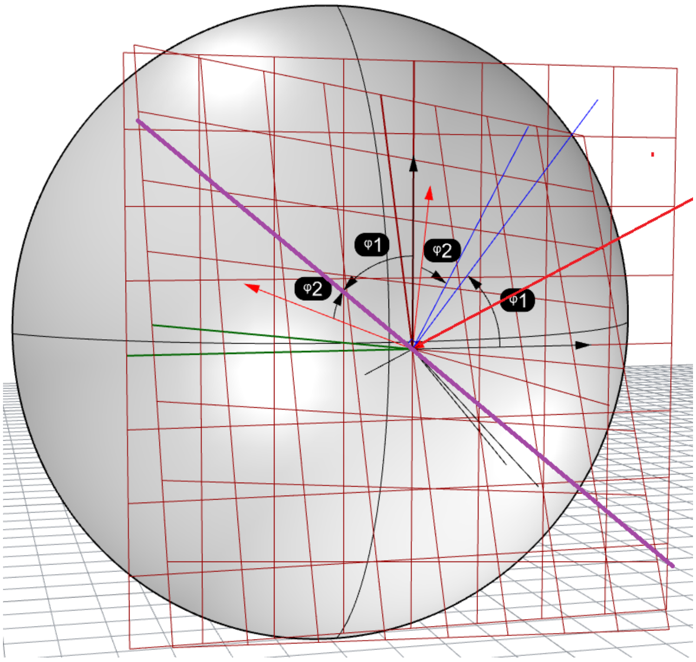

Figure 3 – Demonstration of the curvature tensor rotation technique. Both tensors must be rotated so that at least one of their principal radii coincide with the line of intersection of their respective perpendicular planes. φ1 refers to the angle of rotation for the surface curvature tensor. φ2 refers to the angle of rotation for the wave-front tensor.

The rotation of the tensors is achieved using the following equation (4) (note that for as long as the wave-front is spherical, the orientation of the wave-front axis is arbitrary. Rotation is only required for the surface curvature tensor until the second order reflection, at which point wave-front rotation is also needed):

$$[K]' = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} [K] \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix} \tag{4}$$

Once the wave-front and surface tensors have been rotated to coincide on one principal radius each, the remaining radii and the original ray and the normal of the surface all lie in the same plane. The angle between the tangent vectors of the remaining radii can be found. (See figure 4 below) Once the angle Θ between these vectors is known, the wave-front tensor can be modified according to the following equation:

$$[K]_{out} = \begin{bmatrix} g_{11} & -g_{12} \\ -g_{21} & g_{22} \end{bmatrix}_{wave-front} + 2 \begin{bmatrix} g_{11} \sec \Theta & -g_{12} \\ -g_{21} & g_{22} \cos \Theta \end{bmatrix}_{surface} \tag{5}$$

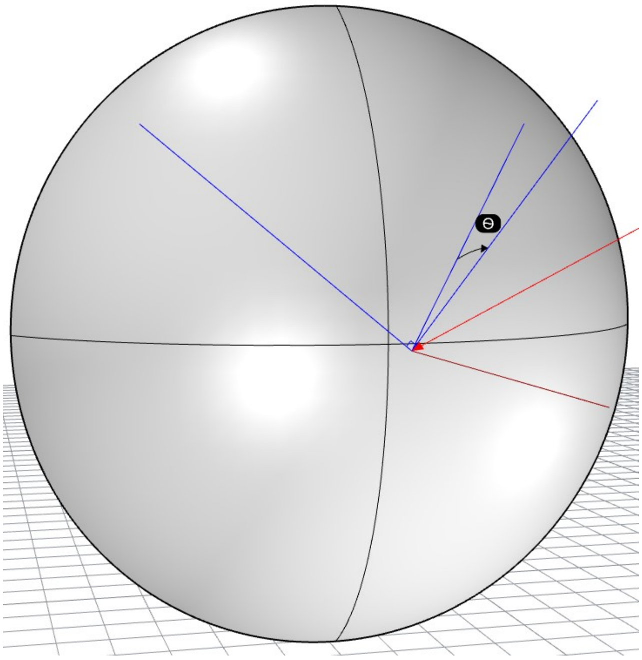

Figure 4 – Demonstration of the curvature tensor rotation technique. Both tensors must be rotated so that at least one of their principal radii coincide with the line of intersection of their respective perpendicular planes. $^{\varphi}1$ refers to the angle of rotation for the surface curvature tensor. $^{\varphi}2$ refers to the angle of rotation for the wave-front tensor.

Before each reflection, the wave-front tensor should be updated to reflect the distance that sound has traveled. Each of the remaining terms should be inverted to the radius form ($r = 1/g$) of the matrix. The incremented distance can then be added to the terms in the primary diagonal of the matrix, and the matrix terms inverted back to curvature form.

$$[K]' = \begin{bmatrix} \dfrac{1}{r_{11}+d} & g_{21} \\ g_{12} & \dfrac{1}{r_{22}+d} \end{bmatrix}$$

- or - (6)

$$[K]^{-1} = [K]^{-1} + \begin{bmatrix} \mathrm{d} & 0 \\ 0 & d \end{bmatrix}$$

Where d is the distance traveled following the reflection from the curved surface, and the inversion indicated in the second equation is a piecewise inversion of the terms of the matrix only (such as $g^{-1} = 1/g$, not a full matrix inversion).

2.4 Model Representation and Geometry Querying

Acoustical analysis via geometrical techniques is still best performed on models composed of polygons, for reasons to do with efficiency. NURBS surfaces, however, allow a very precise representation of curvature, which is a great advantage where the methods discussed in this paper are concerned (without NURBS, curvature would need to be extrapolated from the polygon model, a process that would be fraught with assumptions). For this reason, the implementation in Pachyderm Acoustic (6) (the author's open-source software project) utilizes NURBS based models, with polygon approximations that are produced by tessellating the surfaces. The method described here is performed on the polygon approximations. Each polygon element has curvature information attributed (radius tangents and curvature tensors) which was obtained by querying the NURBS model.

There are a variety of ways to store the information. One could attribute the curvature information to each polygon. If this were the preferred approach, it should be noted that a simple image-source method would be inadequate to find every possible reflection path. Figure 5 below illustrates a convex case in which the image source method would miss a reflection that would exist for the curve represented. Similar cases also exist for concave surfaces. What is needed is a method that can account for the normals that exist on the curved surface, but which lie between the polygons in the polygon approximation.

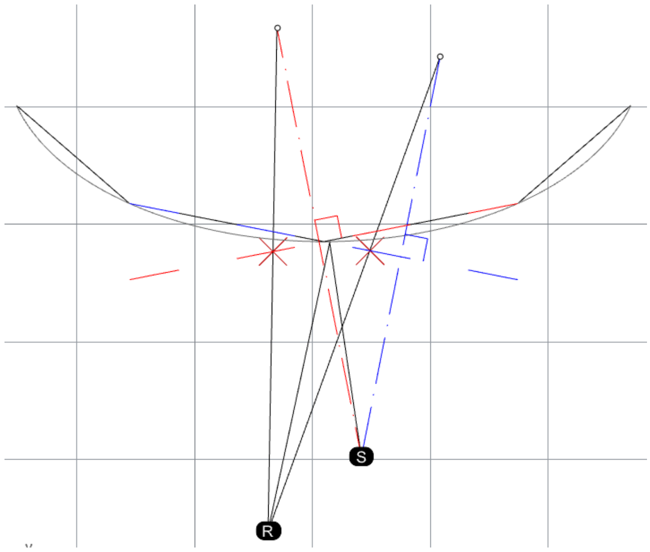

Figure 5 – The image source method alone can not find all reflections from curved surfaces.

Another way to account for the information is to attribute the curvature information to each polygon edge. Using the edges to find specular reflections is simple – 1) Find the midpoint of a direct line from the source to the receiver. 2) Find the closest point on the line through the edge to the previously found midpoint. If the point does not lie on the edge, then there is not a valid specular reflection. If there is, we move on to the next step. 3) Calculate the normalized vector between the midpoint between source and receiver, and it's closest point on the edge. This vector is the normal that would be required to exist on the surface in order for a reflection from the edge to be valid. 4) Compare the supposed normal to the normal for the two polygons connected to the edge. If the vector lies between both polygon normals, then the reflection is valid.

For a convex surface, this point is the entire point of reflection. For a concave surface, the point indicates that there is a region that encapsulates the point which reflects. The creation of the impulse response must account for the full area of the reflection in some way. In the time domain, it must be assumed that the power produced by this point is sustained by the region around it as well.

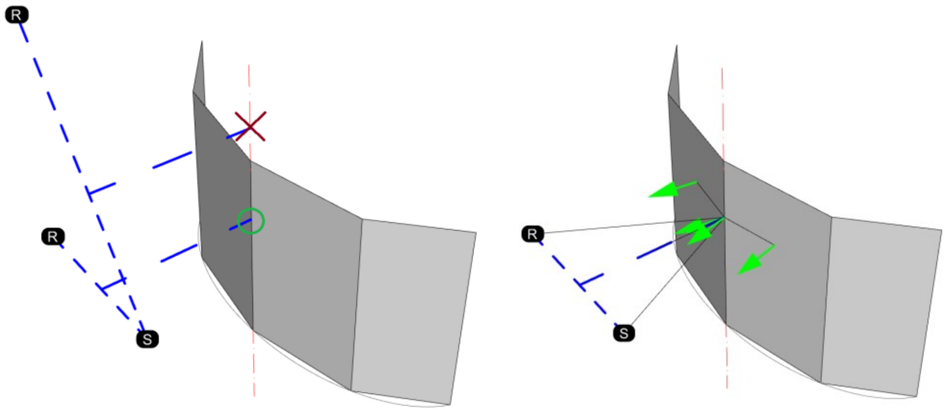

Figure 6 – Edge analysis for finding specular reflections (left) edge closest point operation (right) test for existence of normal between faces.

2.5 Construction of Impulse Responses

In the case of reflection from surfaces with two non-negative curvatures along the principal axes (i.e. flat or convex surfaces), the reflection is very simple, and the full method for calculating it is discussed in the previous sections. This is because the extent of reflecting surface area pertaining to the specular reflection is a point. For the case of surfaces with concave curvature in at least one direction, the reflecting area is larger. In order to fully calculate it, the method in question must account for the entire reflecting area. (see figure 7, below)

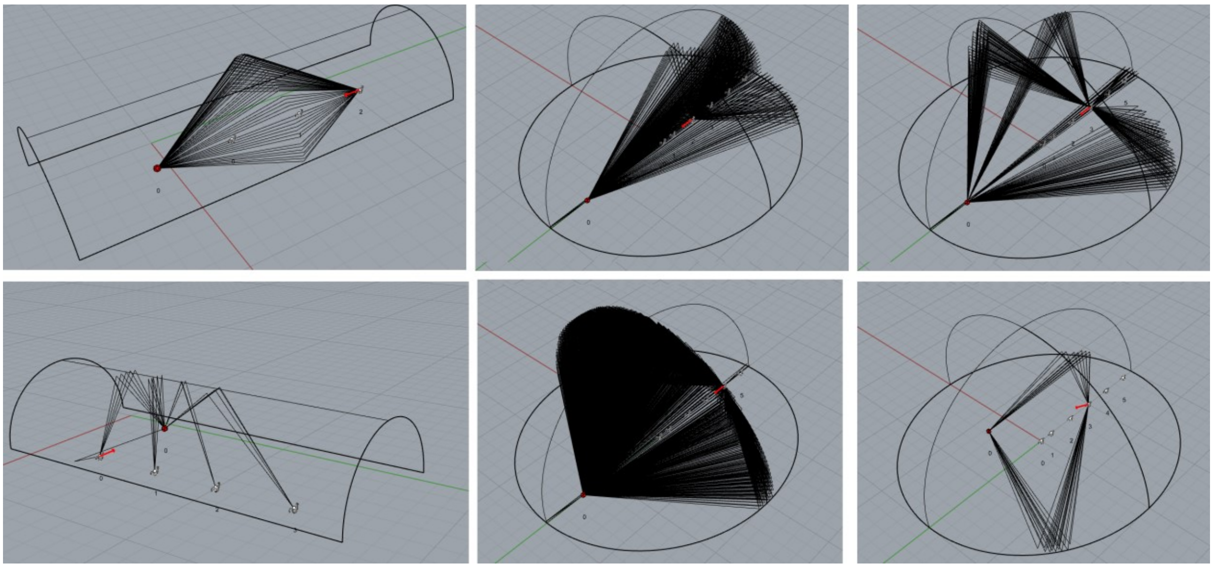

Figure 7 – Reflections found using the algorithm described in section 2.3 for the case of reflections from a surface with negative curvature along one axis and two axes, respectively, taken from several positions.

The full magnitude of the reflection must take into account the area of the surface that contributes to the reflection. The tributary area of each edge must be integrated. The equation below is used to calculate the magnitude of a first order reflection from any given polygon edge on the surface:

$$I_{focus}(t) = \frac{1}{4\pi}\sum_{i=1}^{n} A_t(i)\left|\det[K]_{wave-front}\right| * D(t-\tau) \tag{7}$$

Where n is the number of edges, τ is the time that the reflection from edge i arrives, A_t is the tributary area (or length, if the surface has a negative curvature in only one direction) of the polygon edge, $[K]_{\text{wave-front}}$ is the curvature of the wave-front at the point it reaches its receiver, and $D(t)$ is a distribution function in time, which could be a normal distribution with a width equivalent to the full amount of time the edge contributes, or a distribution calculated based on the geometry of the tributary area. In the case of the latter, care would need to be taken to ensure that no discontinuities exist in the final impulse response.

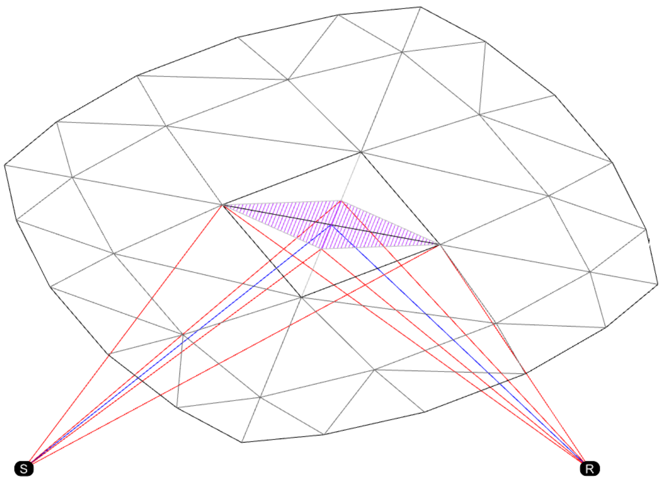

Figure 8 – an illustration of the tributary area of an edge on a polygon forming a larger curved surface. The shortest and longest paths to corners of the tributary area must be known in order to understand the distribution. Note that the tributary area is assumed to extend to the polygon centroid.

In the prediction of sound focusing from curved surfaces, it is not enough to calculate the sum of intensity. The pressure created will vary significantly by frequency due to interference that is a result of the sustained arrival of the reflection (the closer the source and receiver are to the geometrical center of the curved surface, the shorter the reflection will be, and the less interference there will be). The final step is to convert the reflection from intensity to pressure ($p = \rho c \sqrt{I}$) and convolve it with a signal that embodies the phase and power characteristics of the source. One should also include air absorption, surface absorption, and surface scattering in this signal as appropriate. This yields a reflection with the correct time, power, and phase characteristics.

3. CONCLUSIONS AND FUTURE WORK

At present, the most common and accessible techniques for acoustics simulation in the industry falter in accuracy when it comes to reflections from curved surfaces. In this paper, we introduce a new technique that can be used in the context of geometrical acoustics simulation programs.

We have already achieved precise impulse responses for curved surface reflections. Because the great variety in shapes of reflecting areas that can occur on curved surfaces, and the limited space allowed for this paper, we do not include examples of the reflection impulse response, because any example would not be representative. We will show several examples at the lecture associated with this paper.

Future objectives include benchmarking for the new method (and adjustments, as needed), integration with other techniques, such as the BTM edge diffraction technique (the technique used by Pachyderm), and development of a formula for higher order prediction, for accurate prediction of whisper gallery effects and other focusing phenomena for auralization.

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