

Acousto-optic capture of the sound field in a room based on sparse measurement data

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Capturing the spatio-temporal properties of the sound field in a room is valuable for its characterization, as it enables to analyze the specific acoustic space. Current sensing methods typically aim at inferring directional properties of the sound field (i.e., directional impulse responses, power flows, etc.) at a specific location, or a combination of independent locations in the room. This work examines the use of novel sensing principles and reconstruction methods for capturing an acoustic field over a large region of space. This is a challenging problem, as measurements of the sound field over large volumes typically require an unfeasible experimental effort, particularly in large rooms. Specifically, we present an acousto-optic tomography method aimed at reconstructing the sound field from a sparse selection of seemingly incomplete data. A set of wave basis functions is used to interpolate the field, making it possible to predict the sound pressure field over a large spatial aperture. The study presents the measurement principle and experimental results of the volumetric reconstruction of a sound field in a lightly damped room.

Keywords: Room acoustic measurements, acousto-optic effect, sensing, acoustic holography, tomography, compressive sensing, inverse problems.

1. INTRODUCTION

Capturing the spatial properties of the sound field in a room is valuable for its analysis and characterization (1-5). Spatial measurements in rooms are typically performed at multiple independent positions, using sequential measurements with a microphone or with an array of microphones (6), and yield information about the properties of the sound field in the measurement area. However, when it is of interest to obtain phased measurements of the entire sound field in a room to reconstruct it (instead of measuring at multiple independent positions), the problem becomes challenging. Measuring the three-dimensional domain point by point requires extensive measurement effort and it is typically intractable, particularly at mid and high frequencies.

At present, there is a lack of volumetric sensing methods for capturing the entire sound field enclosed in a room. In this study we examine the use of the acousto-optic effect as a volumetric sensing principle. The acousto-optic effect refers to the interaction between sound and light; specifically, changes of the speed at which light travels through a medium in the presence of sound. Sound pressure variations induce local variations in the density of the medium, which in turn change the speed of light, and thus the refractive index.

The interaction between sound and light was theoretically predicted in 1922 (7) and first observed experimentally in 1932 (8-9). Since the 60s, the acousto-optic interaction principle has been extensively used in optics in order to manipulate light using sound (10-11), giving rise to a plethora of acousto-optic applications and devices. In acoustics, the sound-light interaction has been used to sense ultrasound fields (12-13) as well as sound fields in the audible frequency range (14). More recently, laser Doppler interferometry has been proposed to measure changes of the refractive index along beams of laser light (15-17), enabling the use of tomographic techniques to visualize three-dimensional sound fields (18-21). Tomographic techniques based on optical measurements have typically been restricted to controlled laboratory set-ups, partly due to limitations associated with the reconstruction methods employed.

In this study we present an acousto-optic tomography method based on laser Doppler

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interferometry, as the approach is inherently spatially extended, and can be valuable for performing volumetric measurements of the sound field in a room. Existing acousto-optic tomography approaches have fundamentally been used under laboratory conditions, due to technical challenges (which we are also subject to in this study), and partly due to the reconstruction methods employed. Conventional tomographic reconstruction methods, such as the *filtered back-projection* (22), require a uniform scanning of the sound field in parallel beams, at regular rotation intervals, which is a strong limitation for its general in-situ applicability in acoustics. In this study we propose a so-called *algebraic* reconstruction method which does not require a particular measurement configuration or sampling scheme, and which uses a set of wave functions to interpolate the sound field (23-24). The approach is formulated as a wave reconstruction problem, which makes it possible to estimate the sound field over the whole domain, enabling to predict the entire acoustic field in the room, i.e., sound pressure, particle velocity and sound power flows. The sound field can be scanned with arbitrary beams and reconstructed thereby, provided that the overall spatial coverage is sufficient. The method is potentially suitable for performing measurements in real rooms and enclosures, such as auditoria or performing spaces. In the present study we focus on the reconstruction of the sound pressure field. We present the methodology and experimental results of measurements performed in a lightly damped room.

2. THEORY

The refractive index of a transparent medium, n , varies as a function of its density ρ as $n = \rho K + 1$, with K being the Gladstone-Dale constant (25). If sound waves are present in the medium, its density changes with the acoustic pressure as

$$\frac{p+p_0}{p_0} = \left(\frac{\rho}{\rho_0}\right)^\gamma, \quad (1)$$

resulting in the refractive index changing as

$$n = n_0 + \frac{n_0 - 1}{\gamma p_0} p, \quad (2)$$

where γ is the ratio of specific heats, p is the acoustic pressure, and p_0 , n_0 and ρ_0 are the pressure, refractive index and density under static conditions.

Let us consider an arbitrary sound field in a room, and a beam of laser light that traverses the room through the straight path l of length L . The light is emitted by a transmitter, backscattered on the boundary or wall, and eventually collected at a photodetector (see Fig. 1). The resulting phase of the light beam ϕ is determined by the optical path

$$\phi(L, t) = k_l \int_0^L n(\mathbf{r}, t; l) dl. \quad (3)$$

This phase can be sensed by means of an optical interferometer such as a Laser Doppler Vibrometer (LDV). The output of a LDV interferometer is proportional to the change of the phase of light

$$V(L, t) = \frac{1}{k_l n_0} \frac{d\phi}{dt}, \quad (4)$$

so that the eventual vibration measured with the LDV is

$$V = U_w + \frac{n_0 - 1}{\gamma p_0 n_0} j\omega \int_0^L P(l) dl, \quad (5)$$

which consists of the vibration of the wall U_w and a second term due to the acousto-optic effect. When the walls of the room can be regarded as rigid, which is common for hard room boundaries (walls and floors) at mid and high frequencies, the measurement output V contains only the acousto-optic effect.

Equation (5) shows that the measured signal V is proportional to the line integral of the sound pressure along the optical path l , i.e., the sound field projected along the path. It follows that if sufficient projections of the sound field are measured, e.g., scanning the sound field in multiple directions, a tomographic image of the wave field can be obtained, which contains the spatial properties of the sound field. Existing reconstruction methods mainly rely on the *filtered back projection* algorithm. This algorithm is based on the Radon transform of the sound field – i.e., the projection of the field, through line integrals from every direction, onto the k -space domain (22).

Transform-based techniques lead to strong reconstruction artifacts when the projections are not uniformly distributed or not enough projections are available, limiting the applicability of acousto-optic tomography for in-situ measurements.

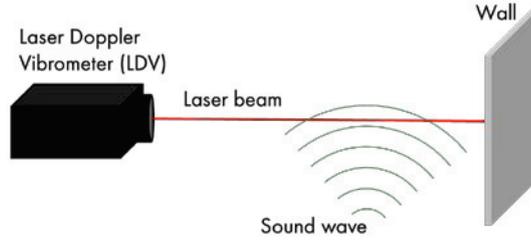


Figure 1 – Measurement diagram of the acousto-optic interaction using a Laser Doppler Vibrometer.

In this study an alternative reconstruction method is proposed, where the field quantity (the sound pressure) is expanded into a plane wave basis of unknown amplitudes X_n ,

$$P(\mathbf{r}) \approx \sum_{n=1}^N X_n e^{j\mathbf{k}_n \cdot \mathbf{r}} . \quad (6)$$

Inserting the wave model into Eq. (5) the measured output is

$$V = \sum_{n=1}^N X_n \frac{n_0 - 1}{\gamma p_0 n_0} j\omega \int_0^L e^{j\mathbf{k}_n \cdot \mathbf{r}} dl , \quad (7)$$

where the unknown amplitudes of the plane waves are X_n and the line integral $\int_0^L e^{j\mathbf{k}_n \cdot \mathbf{r}} dl$ accounts for the projection of each wave in the expansion basis. It is assumed that the wall vibration is sufficiently small to be disregarded (24). Measuring with m projections gives rise to a linear system of equations that can be written algebraically as

$$\mathbf{v} = \mathbf{H}\mathbf{x} , \quad (8)$$

where each element of the transfer matrix \mathbf{H} is

$$H_{nm} = \frac{n_0 - 1}{\gamma p_0 n_0} j\omega \int_0^{L_m} e^{j\mathbf{k}_n \cdot \mathbf{r}} dl_m . \quad (9)$$

This constitutes an inverse problem, where the unknown amplitudes of the plane wave basis \mathbf{x} are to be estimated from the set of measured data \mathbf{v} . Once the amplitudes of the waves are recovered, the sound field can be reconstructed (not only the sound pressure, but also the particle velocity – through Euler's equation, and the sound intensity).

A classical approach to solve the system of equations (8) is to estimate the coefficients \mathbf{x} via a regularized pseudo-inverse,

$$\tilde{\mathbf{x}} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \beta^2 \mathbf{I})^{-1} \mathbf{v} , \quad (10)$$

which yields the least-squares (LS) solution in which the wave amplitudes have minimal energy. This estimate is not always the most appropriate choice, as it minimizes the energy of the solution and is ill-suited to cases where the available data is limited (26-28). The Compressive Sensing (CS) framework is a powerful alternative, which promotes sparse solutions to the problem, i.e., solutions with minimal number of non-zero elements in $\tilde{\mathbf{x}}$ (few plane waves). The approach makes it possible to overcome the traditional sampling limits and reconstruct signals from seemingly undersampled data (26-27). The CS solution is found by solving the optimization problem

$$\tilde{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{H}\mathbf{x} - \mathbf{v}\| \leq \sigma^2 . \quad (11)$$

Once the amplitudes of the plane waves have been estimated, the sound field can be reconstructed by directly evaluating Eq. (6) in the chosen reconstruction domain,

$$\tilde{\mathbf{p}} = \mathbf{W}\tilde{\mathbf{x}} , \quad (12)$$

and each element of the matrix \mathbf{W} is a plane wave at any reconstruction point \mathbf{r}_m , $W_{mn} = e^{j\mathbf{k}_n \cdot \mathbf{r}_m}$.

3. EXPERIMENTAL RESULTS

To test the applicability of the method, an experimental test is conducted in a lightly damped room at the Technical University of Denmark. Recent articles (23-24) present studies of the method based on simulated results, as well as experiments evaluating the applicability of the method, including the assumption that the walls are sufficiently hard as to consider them rigid. The focus of the present experiment is to test the ability of the method to capture a sound field in the volume of a room at high frequencies ($> 1\text{ kHz}$).

The room used in the experiments is approximately rectangular, of dimensions $3.3 \times 4.5 \times 3\text{ m}^3$ and a reverberation time of $T_{60} = 2.2\text{ s}$ (the room is shown in Fig. 2). It is relevant to note that there are scattering elements in the room such as connection boxes and a slit in the ceiling that makes the modal shapes deviate from the analytical cosine shape of a perfectly rectangular room (especially at high frequencies). The room is driven by a dodecahedron source located at position $(x, y, z) = (1.3 \times 1.4 \times 1.5)\text{ m}$. A Polytec scanning LDV (model PSV-400) is used for the measurements. The laser is placed at 7 different positions, and at each position a series of 70 to 75 lines are measured by the scanning head automatically, as shown in Fig. 3 (left). Between each of the 7 positions the laser was moved manually. The total number of projections is 517, covering a total area of 5.8 m^3 , for which the total measurement time is approximately 10 hours. It is estimated that the reconstruction with the proposed method is valid up to about 1.5 kHz. For reference, according to the Nyquist sampling limit, it would have required approximately 4.000 microphone measurements to sample the sound field at this frequency.



Figure 2 – Experimental setup. Left: 360-degree view of the room in which the measurements were performed. Right: Scanning laser vibrometer used to perform the measurements.

Figure 3 shows the result of the reconstructed field in the room at 1.2 kHz. At frequencies above 700 Hz it was verified that the output of the laser vibrometer due to the acousto-optic interaction was larger than the vibration velocity of the wall, as indicated in Eq. (5) and addressed in Ref. (24). The figure (Fig. 3, left) shows a top view of the projected beams that were measured with the LDV, the reconstructed field based on a Least-Squares (LS) estimate of Eq. (8), as well as the Compressive Sensing (CS) reconstruction. For guidance, the sound field in a perfectly rectangular room estimated based on the analytical Green's function is also shown in the figure. The numerical sound field (Fig. 3, mid-left) can serve for qualitative assessment, rather than as a true reference: the geometry of the room used in the experiment is not exactly rectangular, and the boundary properties, directivity of the source, etc., are not accounted for in the numerical Green's function. The quantitative accuracy of the method, based on numerical simulations (23) and experimental results (24) has been examined previously, including a comparison with measured reference sound fields at low frequencies (24). Quantitative assessment of the reconstruction is not presented here (it is not trivial to measure a reference true sound field at 1.2 kHz) and is pending for future work.

The results of the reconstruction (Fig. 3) indicate that the sound field can be reconstructed from the measured data. In the area where the measurements are performed there is good agreement between the LS and the CS reconstruction (i.e., near the measurement domain). In the region of the room that has not been measured, the reconstruction with the LS method is poor, and the overall level of the reconstructed sound field is underestimated due to the plane wave coefficients (with LS) containing minimal energy. Contrarily, the CS estimation seems to preserve the overall levels of the sound field in the room and provides a better estimate when the reconstruction is performed further away from the measured data. This is in good agreement with results reported in the literature (29-30).

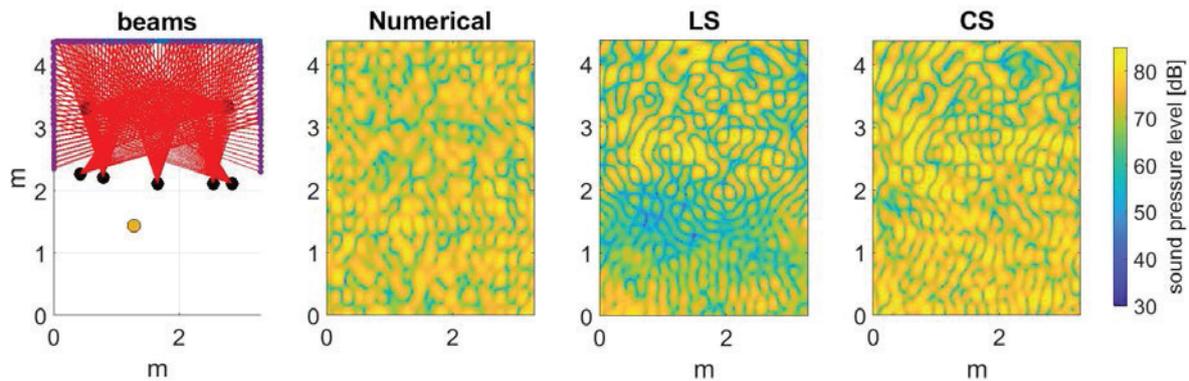


Figure 3 – Reconstruction of the sound field at 1.2 kHz. Left: Diagram indicating the projections in which the sound field is measured with the LDV. The position of the sound source is indicated by the yellow asterisk. Mid-left: Theoretical sound pressure for a perfectly rectangular room, estimated numerically by evaluating the analytical Green’s function. Mid-right: Reconstruction of the sound field using a regularized Least-Squares (LS) estimate [Eq. (8)]. Right: Reconstruction of the sound field using a solution estimate with Compressive Sensing (CS) as in Eq. (11).

Figure 4 shows the histograms of the reconstructed Sound Pressure Levels (SPL) in the room, as well as the SPL from the Green’s function, and the theoretical probability density function of a diffuse field following the random wave model (assuming interfering coherent waves) – see Refs. (31-32). As expected, the numerical Green’s function agrees with the theoretical probability density function, as we are well above the Schroeder frequency of the room. It is apparent that the CS solution follows well the overall statistical distribution, whereas the LS is the least representative one, as the propagation away from the measurement aperture is biased, leading to an underestimation of the acoustic energy in the sound field, and heavier tails in the distribution.

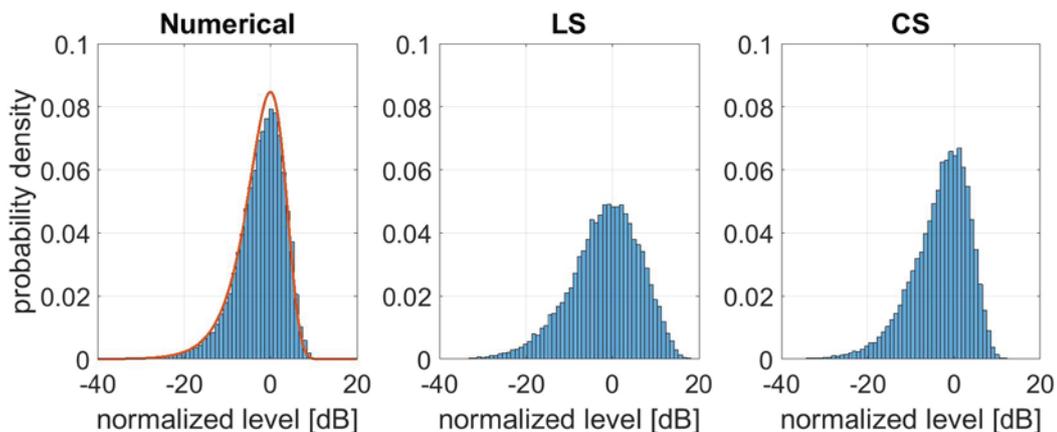


Figure 4 – Statistical distribution of the reconstructed sound fields (histograms of the sound pressure level [dB SPL] in the room normalized to their most probable value). The theoretical probability density function of the sound pressure level in a coherent-wave diffuse field (31) is shown by the solid red line. Left: Numerical Green’s function of the sound field in a perfectly rectangular room. Center: Least-Squares reconstruction. Right: Compressive sensing reconstruction.

It should be noted that the experimental acquisition of the sound field presented in this study is not exempt of challenges: some are of practical nature such as the vibrations of the instrument induced by the acoustic field (the LDV is not designed for this type of measurements but rather to measure vibrations), errors in the estimation of the optical path lengths, etc. There are also more fundamental limitations, such as the lower sensitivity to sound pressure at low frequencies, the fact that the value of the projection integral fluctuates about zero, etc. In the present methodology, the measurements require careful experimental design. It is the author’s view that these challenges are important for the suitability of the method to real in-situ applications and tend to be understated in the literature.

4. CONCLUSIONS

In this study, an acousto-optic tomography method is presented to capture the acoustic field over a large volume inside a room. The method exploits the acousto-optic interaction to sense the acoustic field, and projects the measured data onto a plane wave basis that is used to obtain a volumetric image of the sound field. The sensing principle alleviates the experimental effort required to sample the sound field at high frequencies over a large volume (using conventional transducers). The proposed reconstruction method has the significant benefit of not requiring regular scanning geometries (unlike classical tomographic methods), making it suitable for capturing the sound field in real rooms and enclosures, such as auditoria or performing spaces. At present there are challenges associated to noise levels and sensitivity of the sensing principle, which should be addressed in further work.

ACKNOWLEDGEMENTS

The authors would like to thank Salvador Barrera Figueroa, from the Danish National Metrology Institute (DFM) for valuable discussion and insight, as well as for lending the LDV for the experiments. This work was supported by the VILLUM Foundation, under grant number 19179 for the project ‘Large-scale Acoustic Holography’.

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