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Progresses in the shaping of curved surfaces in concert halls

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ABSTRACT

In concert halls, single convex surfaces are considered as to be acoustically safe elements because they spread sound energy. Whereas single concave surfaces are often unfairly accused of the worst acoustic defaults in concert halls. While simple curvatures are easily predictable from an acoustic point of view, what happens with surfaces which are not concave, nor convex – but which are both simultaneously?
This type of saddle-like surfaces as described in Gaussian terms, are surfaces with negative curvature, in other words, surfaces whose two principal curvatures are of opposite directions.
Performing an acoustic optimisation of complex curved geometries might be challenging. The HK analysis uses the Mean and the Gaussian curvatures (H and K respectively) to undertake a classification of surfaces at a differential level; allowing a closer look to the sound focusing potential of every point on a surface.
However, the HK classification misses all the intermediate curvature radii between the two principal ones. As a complement to HK, Differential Curvature Analysis is run through by the author as a mean of unveiling all the curvature radii of a surface.
Keywords: Concert Halls, Curved surfaces, Focusing.

1. INTRODUCTION: Curved surfaces and acoustics

Acoustic engineers shape sound by shaping surfaces. Fortunately, halls never look the same, which stimulates architects and acoustic engineers to obtain excellent rooms with diverse new typologies or to creatively revisit traditional typologies and shapes.

Years ago, acoustic engineers reached the stage of predicting sound reflections off flat surfaces, sound diffusers and even single radius surfaces has become easily predictable by hand. However, to study more complex surfaces, acoustic engineers tend to apply sound simulation techniques as raytracing (analysing geometry trough acoustic simulations). Acoustic simulations have become cheaper, more flexible and can even be performed in real time through many different approaches as presented in the author's earlier article (1).

This article is about geometry—indeed, what if instead of understanding complex surfaces by indirect methods (acoustic simulations), we started by understanding geometry first?

Shaping a surface while using real-time raytracing even if fast, is still about loops of trial and error. Therefore, this article is about preventing shooting sound rays too soon. It is only once the geometry is understood that a modern real-time design and analysis should be used in order to complete the fine-tuning of the surfaces.

2. DIFFERENTIAL GEOMETRY

By definition, a surface is a two-dimensional collection of points located in the three-dimensional space. The properties of surfaces can be classified into two sorts, the global properties and the local properties: A global property is true for the whole surface space, and in contrast, local properties are hold in a small neighbourhood of a point on a surface, hence the curvature is a local property (2). Local properties are studied by Differential Geometry and will be discussed in this paper.

2.1 Single curved surfaces

The simplest surfaces are curved in one direction with a single radius of curvature. They are basically the extrusion of an arc. Depending on the curvature direction, they are divided into convex or concave surfaces. Because the curvature is equal through the whole surface, we can consider it as a global surface or as a single curved surface, to make it simpler.

Single curved surfaces are easily predictable. The main parameter to look at is the radius of curvature. On the one hand, for convex surfaces, the radius of curvature should be large enough to avoid excessive weakening of sound reflections' energy. On the other hand, in the case of concave surfaces, the radius of curvature should be either large enough so the focusing point lies far beyond the audience or small enough to produce a sound focusing nearer to the surface. In this last case, if the focusing happens too soon, the sound reflection might be too weak when reaching the audience. Rindel (3) proposes ΔL_{curv}, an efficient approach to estimate the attenuation of sound reflections from curved surfaces. This approach, as implemented by Wulfrank (4) allows to visualise sound amplification by curvature in 2D.

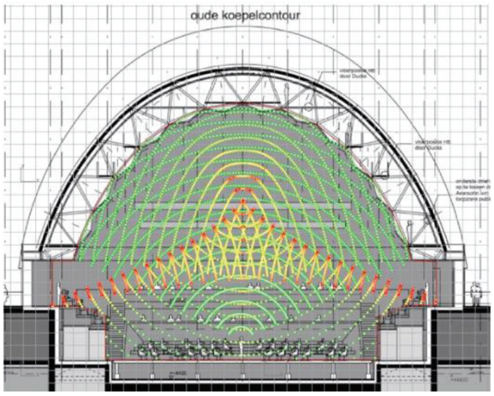


Figure 1 : Example of a ΔLcurv analysis in 2D, as proposed by Wulfrank. From green to red, the colour scale indicates the strength of the sound amplification due to the curvature.

2.2 Multidimensional curved surfaces, HK classification

In this text, the term multidimensional curved surfaces refers to those surfaces whose properties can solely be studied locally—at a differential level, there are no global parameters that can characterise the whole surface. The main relevant property for acoustics is curvature, which is indeed quite complex a notion to explain in a few words (5). The culmination of the studies about curvature is the theorem of Gauss which can be calculated based on quantities which can be measured in the surface itself (5).

The local properties are determined by the two Principal Curvatures at a point, k_1 (maximum radius of curvature) and k_2 (minimum radius of curvature). From these two differential measurements, two main parameters describing curvature are calculated: Mean curvature H and Gaussian curvature K. Equations (1) and (2), describe H and K, both calculated using the two principal curvatures k_1 and k_2.

$$K = k_1 k_2 \quad (1)$$

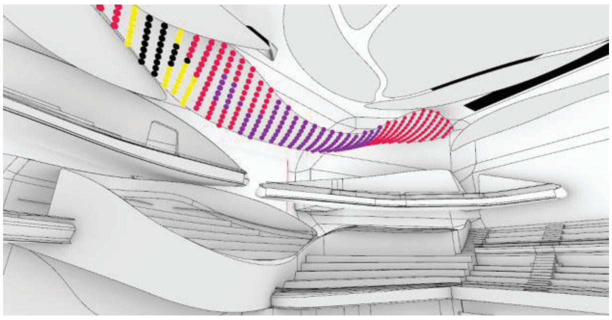
$$H = \frac{(k_1 + k_2)}{2} \quad (2)$$

In these equations, the sign of the Principal Curvatures is meaningful:

- $K < 0$: Negative curvature, the surface is saddle-shaped.
  - Both curvatures have opposite signs.
  - These surfaces have the potential to create sound focusing. An in-detail analysis is required to determine their focusing effects. These are Saddle Ridge, Minimal surface and Saddle Valley (Saddle Valley). To improve readability, *Saddle Valley* will be written in yellow and black.
- $K = 0$: Zero curvature, the surface is flat in at least one direction.
  - One of the main curvatures in 0, which is the case of the single curved surfaces.
  - Both curvatures are 0, which means they are planar surface. Three types, Ridge surface, Flat surface and Valley surface.
- $K > 0$: Positive curvature, the surface is bowl-shaped.
  - Both curvatures point to the same direction—equal signs.
  - Both concave, they describe a dome or Pit Surface—in this case, the whole surface is sound focusing.
  - Both convex, they describe a Peak surface—in that case, the surface is diffusing energy; sound focusing is not possible.

Combining the information extracted from the Mean curvature and the Gaussian curvature, an HK classification of eight surface types is offered by Suk M. (6).The classification as shown in Table 1, has been coloured to link each kind of surface with the 3D analysis that is explained later on.

	$H < 0$	$H = 0$	$H > 0$
$K < 0$	Saddle Ridge	Minimal Surface	Saddle Valley
$K = 0$	Ridge Surface	Flat Surface	Valley Surface
$K > 0$	Peak Surface	None	Pit Surface



(left) Table 1 - Eight surface types based on the HK surface classification (6).
(right) Figure 2 - HK analysis of a *nuage* in the Philharmonie de Paris.

An example of these kinds of surfaces are the *nuages* (ceiling reflectors) of the Philharmonie de Paris, Figure 2. These are multidimensional curved surfaces made of a combination of several types of HK surfaces. Figure 2 highlights the HK analysis on one of the *nuages*. It shows that the centre of the *nuage* (Peak surface) will never focus, whereas the sides can potentially focus at some stage (Saddle Ridge surface)—still, this does not necessarily mean that the focusing is happening near the audience and that it is therefore noticeable.

Figure 2 analysis is performed in Rhino3D using NURBS, coded in Python and RhinoCommon. The analysis is performed in real time. The efficiency of parallel computation allows running the HK analysis while running additional forms of analysis simultaneously, such as Differential Curvature Analysis (see below), raytracing or ΔL_{curv}, among others.

2.3 Differential Curvature Analysis

In complex surfaces, every differential of the surface contains a number of curvature radii, which can be very high. The HK classification only takes into account the two principal curvatures, whereas many others might exist. The author discusses the Differential Curvature Analysis as a method to measure if not all, but most of the curvature radius contained in a differential surface.

What might happen is that the maximum curvature creates a focusing far beyond the audience, and the minimum creates a focusing close enough to the surface to not be perceived as a focusing effect by the audience. The HK methodology might be missing some intermediate curvature radius whose focal point can be a problem.

The Differential Curvature Analysis is suggested by the author as an addition to the HK method. It has been implemented in Rhino 3D, using Python based on RhinoCommon. This method does not evaluate the strength of the sound focusing because it is neither time nor source dependent. It works as follows, cf. Figure 3:

1. For several surface differentials (marked with HK-coloured dots), the curvature radius is measured radially in a large number of radial directions (in addition to the principal curvatures as for HK analysis, many additional curvature radii are measured).
2. Every measured radii correspond to a circle. The centre of each found circle is displayed as a blue dot. In the case of saddle surfaces ($K<0$), one dot on each side of the surface is shown indicating opposite-sign radii of curvature at the very same location.
3. A line between the analysed local point and the centre point is created. For each local point, several lines will reveal several radii of curvature, in which a focusing might take place.
4. For the shake of readability, it allows filtering a certain range of curvature radii. Here, only the radius of curvature whose focal point is near the audience areas can be shown.

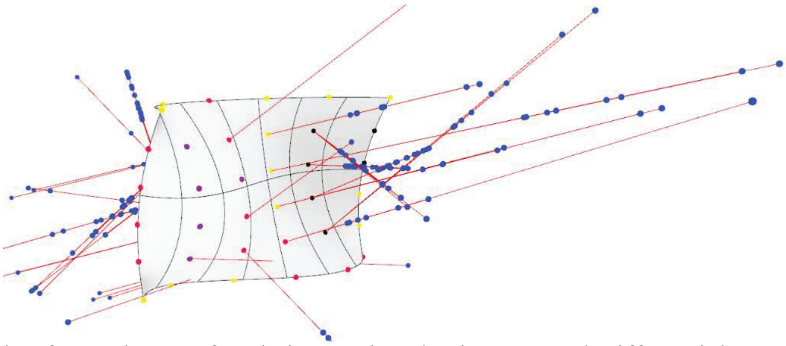


Figure 3 - Example of a random surface being analysed using HK and Differential Curvature Analysis simultaneously.

3. Multidimensional curved surfaces in concert halls, case studies

3.1 Balcony Fronts, the Saddle Ridge

The design of balcony fronts is key in the architectural language of a concert hall. This importance is due to their visually predominant role from the audience's perspective. This is well-known by architects and this is the reason why historically balcony fronts have been the topic of ornamentation and detailed architecture.

Nowadays, the complexity of a balcony front goes beyond ornamentation and is often shaped by acoustics and sight lines. An example of a subtle and elegant acoustic-caused shape is the balcony fronts at the Oslo Opera House, which showcase a diagonal cut separating the acoustically efficient area from the rest.

In concert halls, balcony fronts—especially at the back of the parterre and at choir balconies—are often curved in plan to reinforce a visual intimacy. The centre of these curves is often the conductor's position. This is the case in the Anneliese Brost Musikforum Ruhr, where both balconies are concave in plan, creating a strong focusing directed straightforwardly at the conductor's position and at the centre of the stalls (9).

If the balcony front would have been extruded vertically, it would have been a surface with Gaussian curvature $K = 0$, being a Valley surface. In that case, the only curvature would have been concave in plan and would potentially be risky for good stage acoustics and for the audience´s listening in the parterre.
To solve the potential sound focusing effects, a convex section was designed to create a Saddle Ridge surface, $K < 0$, Figure 4. This convexity was calculated to spread the sound focussing vertically to diminish its negative effect on the sound homogeneity among listeners (9).

Figure 5 shows the Differential Curvature Analysis applied to the same Saddle Ridge-type balcony front of the Anneliese Brost Musikforum Ruhr. At the balcony front, the multiple red lines are the found curvatures and the blue dots their centres. As shown, some red lines contain two blue points, which means that on these blue points, several curvature radii are found at different locations.

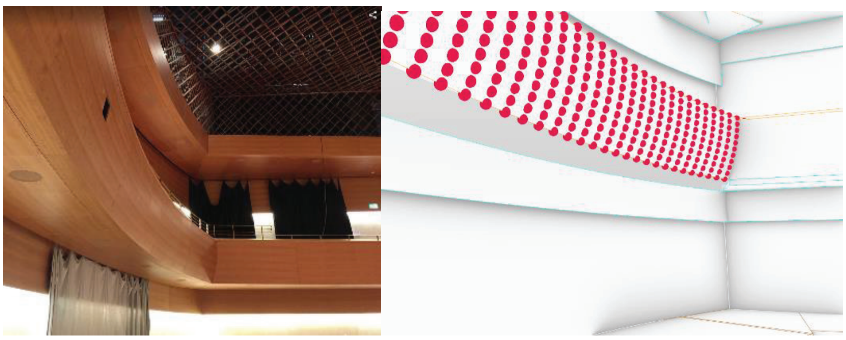


Figure 4 - The Anneliese Brost Musikforum Ruhr balcony front correspond to a Saddle Ridge surface (large concave radius in plan and a small convex radius in section).

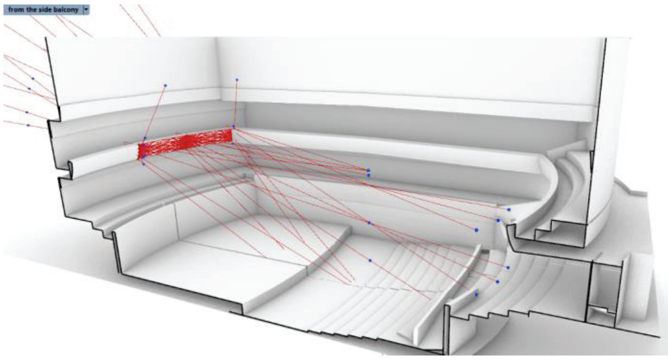


Figure 5 - Differential Curvature Analysis applied to the rear balcony front of Anneliese Brost Musikforum Ruhr.

3.2 Lateral reflectors, the Saddle Valley

Théâtre de Carouge's main hall hides multidimensional curved surfaces. In this hall, the main early reflections are provided by the side walls. After several iterations, the walls were parametrically designed on the basis of a series of superposed, vertically inclined, wood blades. The three main parameters for those continuous panels were: 1) curvature in plan, 37m of radius, 2) a decrease in height depending on how far into the hall and close to the audience the panel was, and 3) a progressive and individual tilting of each reflector panel (1).

Figure 6 shows three design possibilities of the higher blade reflector created especially for this research in order to see the implications in the HK classification:

a) The final design of the lateral wall.
b) The curve in plan is extruded vertically. The only curvature is in plan; therefore, it is a Valley surface and it would produce sound focusing.
c) Progressively inclined (an exaggerated version of *a*). In this case, the fact of creating a *loft* through a series of classifying inclined lines creates a convex virtual (very large) radius of curvature. Since the HK classification shows a Saddle Valley (Saddle Valley) surface – it is understood that the virtual radius of curvature is larger than 37m (H>0).
   By mixing a concave curve in plan and a convex curve in section, the focusing effect is slightly mitigated. Since the radius in plan is much smaller than the radius in section, the diffusion created by convexity is not enough to avoid focusing.
d) Small convex curvature in section is applied to *c*; In this case, the radius of curvature in section is smaller than 37m. The kind of surface is saddle-like, but in this case, since the radius in section is smaller than the radius in plan, it produces a Saddle Ridge surface. Due to this considerable difference, the focusing effect would be highly mitigated. For more precise values, ΔL_{curv} analysis can be used.

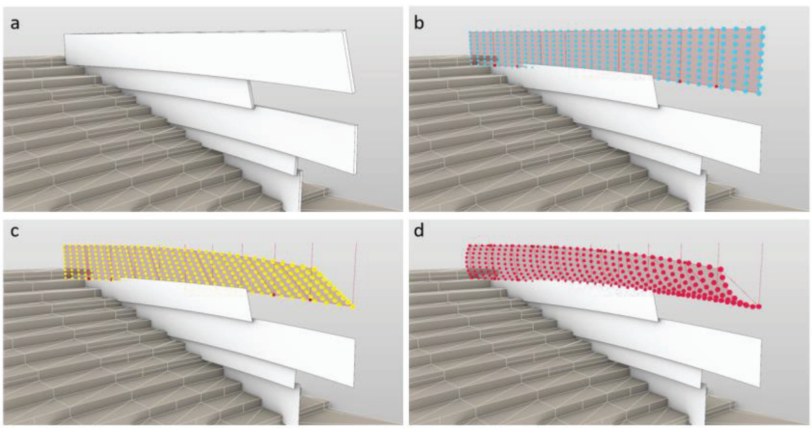


Figure 6 - Three variations of the current design of the Théâtre de Carouge's side walls are shown. a) Current design. b) Vertical extrusion of the curve in plan. c) Exaggerated version of the current design. d) Convex section is applied to *c*.

3.3 Mutating surfaces

In contrast to historical concert halls in which the geometrical complexities are expressed through the ornamentals, the design of modern concert halls often underlays elegantly intricated designs. Some examples of this modern architecture are Harbin Opera House, Guangzhou Opera House and the Philharmonie de Paris.

In this kind of architecture, several architectural and/or acoustic elements might be simultaneously embedded into morphing surfaces with different functions—balconies, sound absorption, diffusion and sound reflectors can be part of a single continuous skin. Several HK typologies can be found through these surfaces.

At the first stage of the design, identifying the HK typologies gives an interesting prediction of the global overall acoustic behaviour. In Figure 7:

- K<0: Red (Saddle Ridge) and yellow areas (Saddle Valley, Saddle Valley) indicate surface areas which are concave and convex simultaneously. While focusing will happen, the localisation and the strength of the focusing should be carefully studied.
- K>0: Black-dotted (Pit surfaces) areas are double concave; therefore, a sound focusing will certainly happen. Purple (Peak surfaces) areas are considered as double convex, therefore one can expect no focusing.

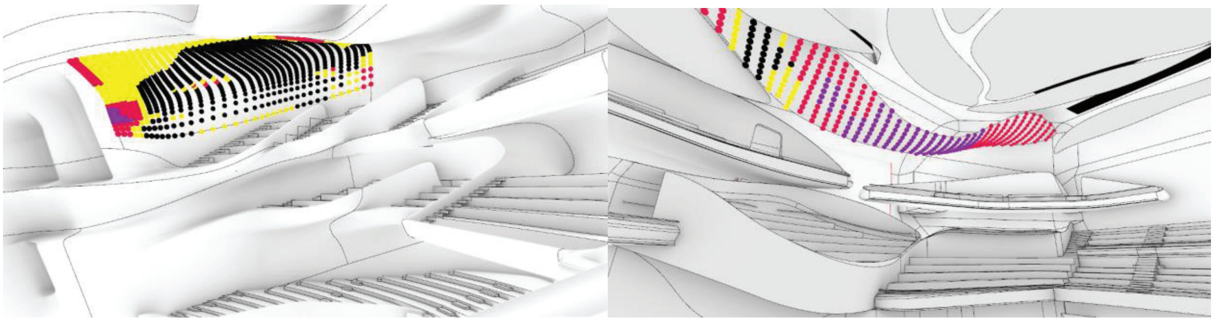


Figure 7 - (left) HK analysis of the side wall of the Guangzhou Opera House. (right) HK analysis of the *nuage* reflector of the Philharmonie de Paris.

Figure 7 shows the HK analysis for portions of the sidewall of the Guangzhou Opera house and of one *nuage* of the Philharmonie de Paris. Both analysis evidence non-homogeneous surfaces which cannot be easily predicted. In this kind of NURBS surfaces, the design parameters are not (at least not only) radii of curvature; in many cases, the position and weight of NURBS's control points are the parameters used to shape the surfaces. A visual inspection of the surfaces is definitely not enough to understand subtle concavities and saddle-like surfaces. Therefore, HK analysis is of great help for *detailing* and *ironing out* this kind of surfaces.

During the design process, due to acoustic or architectural reasons, certain areas will remain concave. Far from being problematic, avoid unfortunate focusing is only a matter of a careful design. Sound focusing might be wanted to happen either:

- Near the curved surfaces: in this case, the sound reaching the audience will be attenuated.
- Beyond the audience: in that case, the sound will gently focus through the audience. Light energy boost that might be appreciated by the audience in the last rows receiving weaker early energy.

Differential Curvature Analysis enables precise design of the distances between the surface and its multiple focal points - allowing to avoid positions near the audience. During a concert, the focal points position depends on multiple factors: source position, source directivity, early reflection patterns… Therefore, designing curved surfaces for a single or few source positions might not consider every possible focusing case.

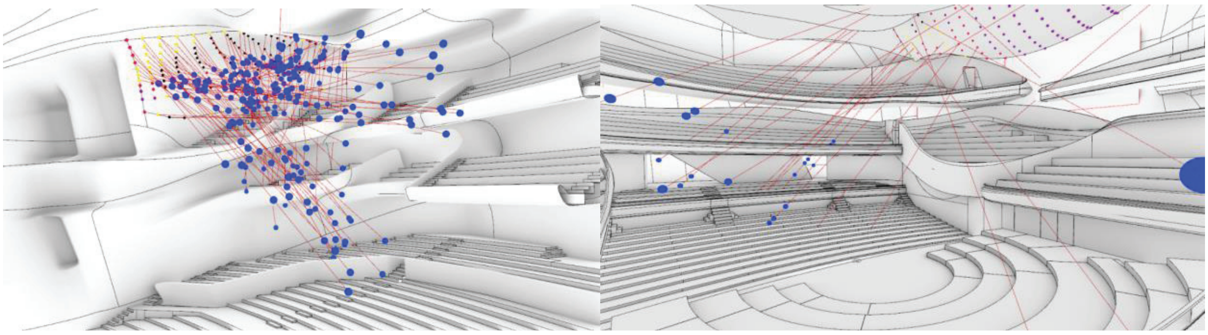


Figure 8 - Differential Curvature Analysis applied to (left) a portion of the side wall of Guangzhou Opera House and (right) a *nuage* of the Philharmonie de Paris (from a different perspective).

Figure 8 shows that in both rooms, for the same areas studied in Figure 7, there are focusing points near the audience. This suggest the necessity to have a closer look at these surfaces to precisely evaluate the focusing risk – action which has surely been undertaken to some extent during the design process.

The 3D models analysed might not be *as built* and these case studies are meant to be an illustration of the capabilities of differential geometric analysis.

4. Conclusions

- Given the strong relation between acoustics and shapes – this paper gives methods to get deeper into the geometrical design of concert halls, instead of shaping hall by trial and error. Raytracing is an indirect method one can use to evaluate surfaces' shape from a sound source perspective. Instead of using indirect techniques to shape surfaces, it is possible to shape from a geometrical perspective first.
- The proposed HK analysis allows classifying surface areas into eight surface typologies, which upon the presence of concavities are classified as *safe* or *to investigate*.
- While the HK analysis is based on the two principal curvatures, by applying Differential Curvature Analysis, most of the hidden focal points are unveiled, and this is especially important for complex curved surfaces.
- Once the surfaces are geometrically acceptable, then acoustic evaluation can be applied.

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